Unique Paper Code : 62354443_OC··

Name of the Paper : Analysis

Name of the Course : B.A.(Prog.)

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

Attempt any four questions. All questions carry equal marks.

1. If a, b are real numbers, prove the following:

(i) If $a + b = 0$, then $b = -a$

(ii) $-(-a) = a$

(iii) $(-a).(-b) = a.b$

Which of the following sets are bounded below, which are bounded above and which are neither bounded below nor bounded above:

(i) $\{-1,\ -2,\ -3, \cdots -n, \cdots\}$

(ii) $\{-1,\ 2,\ -3,\ 4, \cdots (-1)^n n, \cdots\}$

(iii) $\{2,\ \frac{3}{2},\ \frac{4}{3}, \cdots (\frac{n+1}{n})\}$

(iv) $\{3,\ 3^2,\ \ 3^3, \cdots 3^n, \cdots\}$

(v) $\{1,\ \frac{1}{2},\ \frac{1}{2^2}, \cdots,\ \frac{1}{2^n}, \cdots\}$

(vi) $\{-1,\ \frac{1}{2},\ -\frac{1}{3},\ \frac{1}{4}, , \cdots,\ \frac{(-1)^n}{n}, \cdots\}$.

Show that the set of natural numbers has no limit points.

2. Show that the function defined below is not continuous anywhere on the real line.

$$f(x) = \begin{cases} 1 & \text{if x is irrational} \\ -1 & \text{if x is rational} \end{cases}$$

Show that the function $(x) = \frac{1}{x}$, is uniformly continuous on the set $[a, \infty)$, where a is a positive constant.

Show that the set of rational numbers is neither open nor closed.

3. Use the definition of limit of a sequence to establish the following limits as n tends to ∞:

(i) $\lim \left(\frac{n}{n^2+1}\right) = 0$

(ii) $\lim \left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}$

Discuss the convergence of the sequence $\left(\frac{11^{2n}}{7^{3n}}\right)$.

Show that the sequence (x_n) defined by

$$x_1 = a > 0,\ x_{n+1} = \frac{2\,x_n}{1+x_n}\ ,\ \ n > 1.$$

Is bounded and monotone. Also find the limit of the sequence.

4. Give an example of an unbounded sequence that has a convergent subsequence.

Show that the sequence (x_n) defined by

$$x_n = \frac{1}{9} + \frac{1}{13} + \frac{1}{17} \cdots + \frac{1}{4n+5}$$

is not Cauchy.

Calculate the value of $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n}$

5. Discuss the convergence the series Σx_n where x_n is defined as $x_n = \sin\frac{n\pi}{2}$.

Test for convergence of the series $\sum \frac{(2n)!}{(n!)^2}$

Discuss the convergence of the series $\frac{log2}{2^2} - \frac{log3}{3^2} + \frac{log4}{4^2} - + \cdots$

6. Establish the convergence or the divergence of the series whose nth term is:

(i) $\frac{1}{(n+1)(n+2)}$ (ii) $\left(n(n+1)\right)^{-\frac{1}{2}}$ (iii) $\frac{2,\ 4,\ 6,\ldots\ (2n)}{3\ .\ 5\ .\ 7\cdots(2n+1)}$

Let $f(x) = |x|$ for $-1 \leq x \leq 2$. Calculate $L(f;\ \mathcal{P})$ and $U(f;\ \mathcal{P})$ for the following partitions:

(i) $\mathcal{P} =: (-1,\ 0,\ 1,\ 2)$ (ii) $\mathcal{P} =: (-1, -\frac{1}{2},\ 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2)$

Is the function Riemann integrable? Justify.